UNSTEADY TURBULENT FLOW IN A PIPE

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Unsteady turbulent, incompressible fluid flow homogeneous along the length is considered in a circular cylindrical pipe. The formulation of the problem is based on using the Reynolds equations and the energy balance equation of turbulence, which are closed by additional semiempirical relations. Specific computations of the nonstationary turbulent flow based on such an approach are carried out for the case when the discharge in the tube varies with time and performs harmonic oscillations with a finite amplitude around some mean value. The mathematical turbulent flow model under consideration was first tested by comparing the calculated average velocity and turbulent energy distributions with the Laufer test results [1] for stationary flow in a pipe.

Homogeneous flow along the length in a pipe is one of the simplest shear flow examples. In contrast to boundary layer flow, its average characteristics depend only on one space coordinate (the distance from the wall or the radius). Meanwhile, this problem actually contains all the fundamental difficulties, in principle, which are encountered in studying flows with shear. Hence, a study of the specific influence of nonstationarity on turbulent shear flows is most appropriately started with the study of a homogeneous flow in a pipe.

Many investigators have studied the influence of nonstationarity on the turbulent flow in pipes and channels. Thus, J. Daily et al. [2], G. Franke [3], N. A. Panchurin [4], etc., studied such a problem for the case of flow in circular pipes. However, in the theoretical investigations of unsteady flow in pipes known to the authors a semiempirical theory of turbulence was applied which uses the Boussinesq representation of the turbulent viscosity coefficient (sufficiently rough assumptions have been made relative to the latter). Meanwhile, it is doubtful that this theory could reflect those complex turbulence transport and diffusion processes which should hold in a nonstationary turbulent flow well enough. By virtue of the above, it is pertinent in this case to turn to modern statistical models of turbulent flow constructed in application to the transport equations and, in particular, to the turbulent energy transport equations.

Underlying the approach used herein are the ideas of A. N. Kolmogorov, in conformity with which the fundamental statistical characteristics of a turbulent flow can be expressed in terms of the energy and scale of the turbulence. This direction has been developed by V. G. Vager and D. L. Laikhtman [5] and V. G. Levin [6] in application to steady flow in pipes.

E. V. Eremenko used such an approach in studying unsteady flows when analyzing plane-parallel flows in channels. He computed the kinematic characteristics in a plane tube with harmonically varying pressure gradient in [7]. He extracts the diffusion of the pressure fluctuation energy in the equation and inserts an approximate expression for it separately. The coefficient of turbulent viscosity is taken proportional to the so-called turbulent Reynolds number, and the coefficient of turbulent diffusion in conformity with G. S. Glushko [8]. The coefficients of turbulent exchange thereby turn out to be related to the distribution of turbulence energy in space and time. However, it should be noted that the proportionality of the coefficient of turbulent viscosity to the turbulent Reynolds number is not conserved near the wall (low turbulent Reynolds numbers), where the main part of turbulence energy generation occurs.

The coefficient of turbulent viscosity (and the coefficient of turbulent diffusion in terms of it) is taken herein as some function of the turbulent Reynolds number. G. S. Glushko [8] first mentioned such dependences in an analysis of turbulent shear flow in the boundary layer, and he constructed this dependence in the form of a piecewise-smooth function on the basis of numerous experimental results. It is here pro-

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posed to approximate this dependence by some smooth functions with asymptotic y^3 as $y \to 0$, where y is the distance from the tube wall.

1. Fundamental Equations and Additional Relationships

For an incompressible fluid, the Reynolds and turbulence energy e equations for axisymmetric motion in a cylindrical coordinate system (x, r, φ) are

$$\frac{d\langle u_r\rangle}{dt} - \frac{\langle v_\varphi\rangle^2}{r} = -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial r} + \nu\left(\nabla^2\langle u_r\rangle - \frac{\langle u_r\rangle}{r^2}\right) +$$
$$+\frac{1}{r}\frac{\partial}{\partial r}(-r\langle u_r'^2\rangle) + \frac{\partial}{\partial x}(-\langle u_r'u_x'\rangle) - \frac{1}{r}(-\langle u_\varphi'^2\rangle)$$

$$\frac{d\langle u_\varphi\rangle}{dt} + \frac{\langle u_r\rangle\langle u_\varphi\rangle}{r} = \nu\left(\nabla^2\langle u_\varphi\rangle - \frac{\langle u_\varphi\rangle}{r^2}\right) +$$
$$+\frac{\partial}{\partial r}(-\langle u_r'u_\varphi'\rangle) + \frac{\partial}{\partial x}(-\langle u_\varphi'u_x'\rangle) + \frac{2}{r}(-\langle u_r'u_\varphi'\rangle)$$

$$\frac{d\langle u_x\rangle}{dr} = -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x} + \nu\nabla^2\langle u_x\rangle + \frac{1}{r}\frac{\partial}{\partial r}(-r\langle u_r'u_x'\rangle) + \frac{\partial}{\partial x}(-\langle u_x'^2\rangle)$$

$$\frac{\partial}{\partial r}(r\langle u_r\rangle) + \frac{\partial}{\partial x}(r\langle u_x\rangle) = 0$$

$$\frac{de}{dt} = -\frac{\partial}{\partial x}\left\langle u_x'\left(e' + \frac{p'}{\rho}\right)\right\rangle - \frac{1}{r}\frac{\partial}{\partial r}r\left\langle u_r'\left(e' + \frac{p'}{\rho}\right)\right\rangle -$$
$$-\langle u_x'^2\rangle\frac{\partial\langle u_x\rangle}{\partial x} - \langle u_r'^2\rangle\frac{\partial\langle u_r\rangle}{\partial r} - \langle u_\varphi'^2\rangle\frac{\langle u_r\rangle}{r} -$$
$$-\langle u_x'u_r'\rangle\left(\frac{\partial\langle u_x\rangle}{\partial r} + \frac{\partial\langle u_r\rangle}{\partial x}\right) + \nu\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}e +\right.$$
$$\left.+\frac{\partial\langle u_x'^2\rangle}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\langle u_x'u_r'\rangle)\right] + \frac{\nu}{r}\frac{\partial}{\partial r}r\left[\frac{\partial}{\partial r}e +\right.$$
$$\left.+\frac{\partial}{\partial x}\langle u_x'u_r'\rangle + \frac{1}{r}\frac{\partial}{\partial r}r\langle u_r'^2\rangle - \frac{\langle u_\varphi'^2\rangle}{r}\right] - \Delta \tag{1.1}$$

Here t is the time, the x axis is directed along the tube axis, u_x, u_r, u_φ are velocity vector components, p is the pressure, e is the turbulence energy, ρ is the density, ν is the kinematic viscosity (the angular brackets denote average quantities,

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \langle u_x\rangle\frac{\partial}{\partial x} + \langle u_r\rangle\frac{\partial}{\partial r},\ e\,\frac{\langle u_x'^2\rangle + \langle u_r'^2\rangle + \langle u_\varphi'^2\rangle}{2}$$
$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r},\ \Delta \equiv \nu\left[2\left\langle\left(\frac{\partial u_x'}{\partial x}\right)^2\right\rangle +\right.$$
$$+2\left\langle\left(\frac{\partial u_r'}{\partial r}\right)^2\right\rangle + 2\left\langle\left(\frac{\partial u_\varphi'}{r\partial\varphi} + \frac{u_r'}{r}\right)^2\right\rangle + \left\langle\left(\frac{\partial u_x'}{\partial r} + \frac{\partial u_r'}{\partial x}\right)^2\right\rangle +$$
$$\left.+\left\langle\left(\frac{\partial u_\varphi'}{\partial x} + \frac{\partial u_x'}{r\partial\varphi}\right)^2\right\rangle + \left\langle\left(\frac{\partial u_\varphi'}{\partial r} + \frac{\partial u_r}{r\partial\varphi} - \frac{u_\varphi'}{r}\right)^2\right\rangle\right]$$

Let us examine the nonstationary turbulent motion of an incompressible fluid in a circular cylindrical tube of radius R. Let us introduce the following assumptions: there exists axial flow symmetry, the flow is statistically homogeneous along the tube axis (i.e., the average values of the velocity components and the products of their pulsations are independent of the variable x), and the tangential component of the average velocity is zero. It can be considered that these assumptions are satisfied for the flow in a circular tube at sufficiently long ranges from the inlet and exit sections. Under these assumptions, it follows from the continuity equation and the condition of impermeability on the tube wall that $\langle u_r\rangle = 0$. Consequently, the system (1.1) simplifies noticeably and becomes

$$\frac{\partial\langle u_x\rangle}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}r\left(\nu\frac{\partial\langle u_x\rangle}{\partial r} - \langle u_r'u_x'\rangle\right) - \frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x}$$
$$\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial r} + \frac{1}{r}\frac{\partial r\langle u_r'^2\rangle}{\partial r} - \frac{\langle u_\varphi'^2\rangle}{r} = 0$$
$$\frac{\partial e}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}r\left\langle u_r'\left(e' + \frac{p'}{\rho}\right)\right\rangle - \langle u_x'u_r'\rangle\frac{\partial u_x}{\partial r} +$$
$$+\frac{\nu}{r}\left[\frac{\partial}{\partial r}r\frac{\partial}{\partial r}e + \frac{\partial^2}{\partial r^2}r\langle u_r'^2\rangle! - \frac{\partial}{\partial r}\langle u_\varphi'^2\rangle\right] - \Delta \tag{1.2}$$

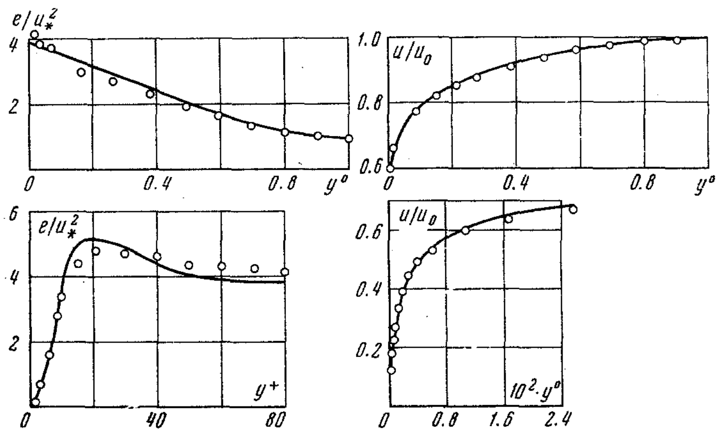

Fig. 1

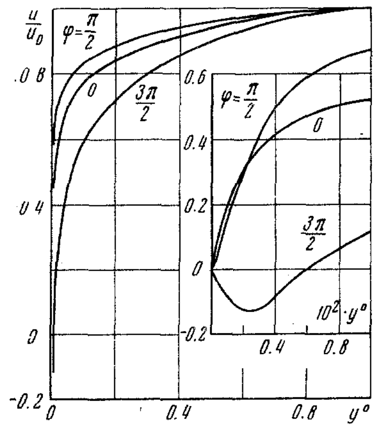

Fig. 2

Furthermore, it can be shown that the pressure gradient $\partial \langle p \rangle / \partial x$ is a function of only the time t. Indeed, by integrating the second equation of the system (1.2) with respect to r we obtain

$$\langle p \rangle - \langle p_0 \rangle + \rho \langle u_r'^2 \rangle + \rho \int_R^r \frac{\langle u_r'^2 \rangle - \langle u_\varphi'^2 \rangle}{r} dr = 0$$

Here $\langle p_0 \rangle = \langle p(R, t, x) \rangle$. Since $\langle u_r'^2 \rangle$ and $\langle u_\varphi'^2 \rangle$ are independent of x, then $\partial \langle p \rangle / \partial x = \partial \langle p_0 \rangle / \partial x$ and $\partial \langle p \rangle / \partial x$ is a function of only t and x. And then there results from the first equation of the system (1.2) that $\partial \langle p \rangle / \partial x$ is a function of just the time.

Now, let us examine the terms describing the work of the viscous stresses. They are represented by the components in the square brackets in the turbulent energy equation of the system (1.2). It is known that the work of the viscous stresses is essential only near the tube wall. By using the asymptotic representation ([9], p. 236) of the energy pulsation component near the wall, it can be shown that the fundamental contribution to the work of the viscous stresses is introduced by the term $(\nu/r)\,\partial(r\partial e/\partial r)/\partial r$. Taking this circumstance into account, let us furthermore neglect the remaining members of the work of the viscous stresses.

Only two differential equations of the system (1.2) are needed for a further analysis of the flow: the first and the third, i.e., the Reynolds equation for the longitudinal velocity and the turbulence energy equation. However, the number of unknowns in these equations exceeds the number of equations. Following G. S. Glushko [8], let us use semiempirical hypotheses to close the system:

1) Momentum transfer is accomplished by gradient type diffusion

$$-\langle u_r' u_x' \rangle = \varepsilon \partial \langle u_x \rangle / \partial r \tag{1.3}$$

2) The transfer of total turbulent energy is described similarly as a gradient type diffusion process:

$$\nu \frac{\partial e}{\partial r} - \left\langle u_r' \left(e' + \frac{p'}{\rho} \right) \right\rangle = D \frac{\partial e}{\partial r} \tag{1.4}$$

3) The process of turbulence energy dissipation is defined by the relationship

$$\Delta = CDe / L^2 \tag{1.5}$$

Here ε is the coefficient of turbulent viscosity, D is the total diffusion coefficient, L is the scale of turbulence, and C is a universal constant.

The three relationships presented do not evidently contain new information, in substance, if only ε, D and L have not been defined. The empiricism of these formulas is due to the mentioned coefficients and the length scale being determined by involving experimental results. It turns out that by being guided by physical considerations, relatively simple dependences for ε, D, and L can be constructed. Thus, A. N. Kolmogorov proposed considering $\varepsilon \sim \sqrt{e}L$. By analyzing the turbulent flows in the boundary layer, G. S. Glushko [8] showed that the coefficient ε can be represented as a function of just the turbulent Reynolds number $Re_t = \sqrt{e}L/\nu$.

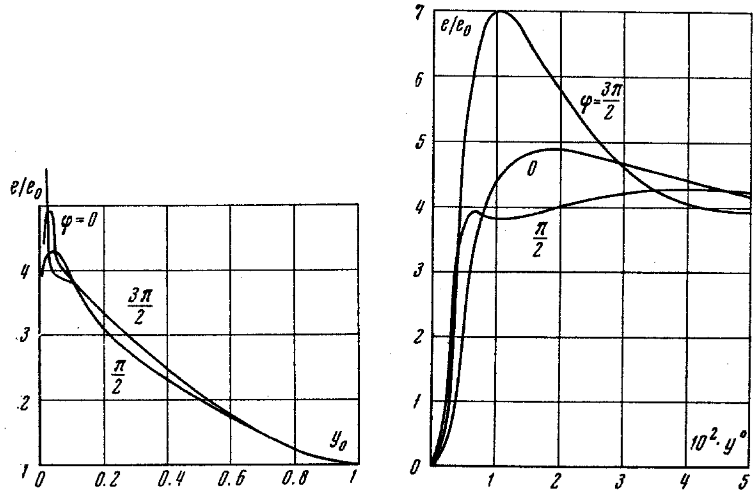

Fig. 3

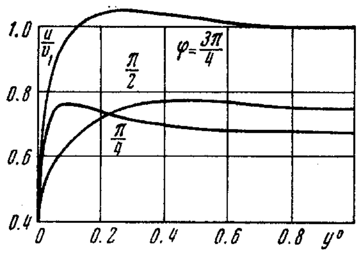

Fig. 4

Let us examine the behavior of the turbulent viscosity coefficient in the near-wall domain in more detail by representing it as a power-law dependence $\varepsilon \sim y^n$, where y is the distance from the wall. As follows from the continuity equation for the velocity pulsations and the condition that they be zero at the wall itself (at $y=0$), the exponent is $n \geq 3$. There are both experimental and theoretical investigations indicating an exponent $n=3$ favorably (see [9, 10], say). This is also verified by a recent investigation [11] of the flow in the near-wall domain, based on the principle of maximum stability.

According to the experimental results of Laufer (see [9]), $e \sim y^2$ in the near-wall domain. Processing of numerous experiments performed by G. S. Glushko [8] yields $L \sim y$ as $y \to 0$. It can hence be concluded that the asymptotic $\varepsilon \to \mathrm{Re}_t^{3/2}$ should hold as $\mathrm{Re}_t \to 0$ (under the assumption that the coefficient ε depends only on the turbulent Reynolds number). Taking account of this circumstance, the dependence $\varepsilon = \varepsilon(\mathrm{Re}_t)$ which G. S. Glushko [8] approximated by a piecewise-smooth function, is approximated herein by a smooth function with the asymptotic $\varepsilon \sim \mathrm{Re}_t^{3/2}$ as $\mathrm{Re}_t \to 0$:

$$\varepsilon / \nu = \alpha \, \mathrm{Re}_t \left[1 - \exp(-\sigma_2 \mathrm{Re}_t^2) + \sigma_3 \mathrm{Re}_t^{1/2} \exp(-\sigma_1 \mathrm{Re}_t^2)\right] \tag{1.6}$$

For $\sigma_1 = 4 \cdot 10^{-4}$, $\sigma_2 = 2.1 \cdot 10^{-4}$, $\sigma_3 = 2 \cdot 10^{-2}$ and $\alpha = 0.2$ formula (1.6) agrees well with the dependence proposed by G. S. Glushko everywhere except in the direct neighborhood of the point $\mathrm{Re}_t = 0$. Let us recall that $\varepsilon \sim \mathrm{Re}_t^2$ as $\mathrm{Re}_t \to 0$ for G. S. Glushko. Since $\varepsilon/\nu = \alpha\sigma_3 \mathrm{Re}_t^{3/2} + O(\mathrm{Re}_t^3)$ as $\mathrm{Re}_t \to 0$ in conformity with (1.6), then the coefficient of turbulent viscosity in the direct neighborhood of the point $\mathrm{Re}_t = 0$ is determined by the constant σ_3. The value $\sigma_3 = 2 \cdot 10^{-2}$ is determined by processing the Laufer [1] experimental results for a circular tube. Let us note that (1.6) yields the asymptotic $\varepsilon/\nu = \alpha \mathrm{Re}_t$ for large Re_t.

Let us also assume that the total coefficient of turbulence energy diffusion is related to the coefficient of turbulent viscosity by the linear dependence

$$D = \nu + m\varepsilon \tag{1.7}$$

where m is a constant coefficient. The reciprocal quantity to m is the analog of the turbulent Prandtl number.

No successful and sufficiently reliable theoretical dependence has yet been found for the scale of turbulence L which would relate it to the other flow characteristics. In this connection, let us approximate the scale L herein by the polynomial expression

$$L / R = l_0 + l_2 (r / R)^2 + l_4 (r / R)^4 \tag{1.8}$$

It is easy to note the analogy between this expression and the known Nikaradze formula for the mixing path length for flow in a tube.

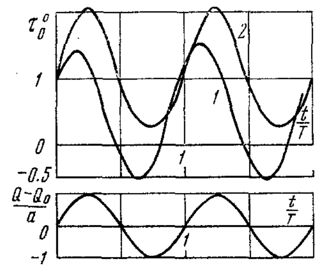

Fig. 5

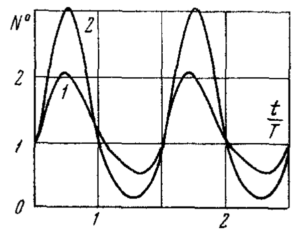

Fig. 6

Two of the constants l_0, l_2, l_4 can be determined by using the empirical dependence for L from [8] for r = R, namely

$$L(R) = 0, \; dL / dr|_{r=R} = -1$$

Let us hence assume that the scale of turbulence L in the neighborhood of the wall in boundary-layer flow and in the flow in a tube are identical. The third constant l_0 was determined from the condition that $\varepsilon/u_*R \approx \alpha\sqrt{eL}/u_*R \approx 0.07$ at $r = 0$. Such a value of ε/u_*R was calculated by Heintze [12] for the middle of a tube according to the Laufer [1] data. Therefore, the following values of the parameters have been obtained: $l_0 = 0.37$, $l_2 = -0.24$, $l_4 = -0.13$. The constants C and m, corresponding to the mentioned values of the remaining constants, equal $C = 3.93$ and $m = 0.4$ in conformity with [8].

2. Formulation of the Problem

Summarizing the above, we arrive at the equations

$$\frac{\partial u}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} r(\nu + \varepsilon)\frac{\partial u}{\partial r} - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial e}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} rD\frac{\partial e}{\partial r} + \varepsilon\left(\frac{\partial u}{\partial r}\right)^2 - CD\frac{e}{L^2} \qquad (2.1)$$

Here the coefficients ε, D and the scale L are determined by the expressions (1.6)-(1.8). To simplify the notation, the averaging signs and the subscript x for the single non-zero component u_x of the averaged velocity vector have been omitted.

The boundary conditions will be the following:

$$\frac{\partial u}{\partial r} = \frac{\partial e}{\partial r} = 0 \quad \text{for} \quad r = 0, \qquad u = e = 0 \quad \text{for} \quad r = R \qquad (2.2)$$

For $r = 0$ the conditions (2.2) follow from the flow symmetry relative to the tube axis, and for $r = R$ from the condition of adhesion to the smooth wall.

Periodic motion in a tube can be examined on the basis of (2.1) and (2.2). In the more general case, the initial conditions

$$u = u(r), \quad e = e(r) \quad \text{for} \quad t = 0 \qquad (2.3)$$

must be appended.

To solve specific problems, the pressure gradient or the discharge as a function of the time must still be given in addition to the mentioned conditions. The discharge

$$Q(t) = 2\pi \int_0^R urdr \qquad (2.4)$$

is considered given herein.

The independence of the discharge from the longitudinal coordinate results from the continuity equation and expresses the fact that the discharge in any section of a rigid tube is identical for an incompressible fluid. Giving the discharge permits elimination of the pressure p from the considerations, which will now generally be among the desired functions.

Indeed, by integrating the first equation of (2.1) under the conditions (2.2) and (2.4) we obtain

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=\frac{1}{\pi R^2}\left(\frac{\partial Q}{\partial t}-2\pi R\nu\left.\frac{\partial u}{\partial r}\right|_{r=R}\right) \tag{2.5}$$

3. Energy Dissipation

Involvement of the turbulence energy equation permits decoding the mechanism of energy conversion in a turbulent flow, as well as calculating the energy loss. Let us integrate the equations of the system (2.1) over the tube section by first multiplying the first equation by u. Adding them, we obtain the integral equation of the total energy balance after several manipulations

$$-Q\frac{\partial p}{\partial x}=2\pi\frac{\partial}{\partial t}\int_0^R\rho\left(\frac{u^2}{2}+e\right)r\,dr+2\pi\int_0^R\left[\mu\left(\frac{\partial u}{\partial r}\right)^2+\rho CD\frac{e}{L^2}\right]r\,dr \tag{3.1}$$

Here the additional condition $\partial e/\partial r=0$ at $r=R$, which follows from the energy equation in the viscous sublayer ($D=\nu$, $e\sim y^2$ as $y\to 0$), is used in addition to (2.2).

The expression (3.1) admits of the following mechanical interpretation. The work of the pressure is expended in changing the kinetic energy of the averaged and pulsation motions and in energy dissipation into heat. A loss in mechanical energy evidently occurs only because of dissipation of the averaged and pulsation motion energy. Therefore, the energy loss in a tube section of unit length is written as

$$N=2\pi\int_0^R\left[\mu\left(\frac{\partial u}{\partial r}\right)^2+\rho CD\frac{e}{L^2}\right]r\,dr \tag{3.2}$$

4. Flow with Periodically Varying Discharge (Numerical Solution)

The system (2.1) with the conditions (2.2), (2.3), (2.5), and the given discharge $Q=Q(t)$ was solved numerically by an implicit six-point finite-difference scheme used earlier to compute the flow in a boundary layer [13]. The numerical solution of the stationary problem found by the method build-up using the given finite-difference scheme generally agrees satisfactorily with the Laufer [1] experimental results. This is indicated by the comparison performed for the Reynolds number $Re_0=2v_0R/\nu=4.232\cdot 10^5$ (Fig. 1). Here v_0 is the average velocity over the cross section. The notation

$$u_0=u(0,t),\quad u_*^2=\left|\nu\partial u/\partial r\right|_{r=R},\ y^\circ=1-r/R,\ y^+=u_*y/\nu$$

is used in Fig. 1, where u_*^2 is the dynamic velocity squared and the solid lines are the computational results. As is seen, good agreement between the calculated velocities and the test results is observed both in the core of the flow and in the near-wall domain.

The computed curves of the turbulent energy distribution are also close enough to the experimental results. In the neighborhood of the wall the numerical solution discloses the characteristic maximum observed in experiment.

Now, let us turn to the nonstationary flow. Let the discharge vary according to the harmonic law

$$Q(t)=Q_0(1+a\sin\omega t) \qquad (Q_0=\pi R^2v_0,\ \omega=2\pi/T) \tag{4.1}$$

Here Q_0 and a are constants. Let us examine the transient because of which the initial stationary motion becomes periodic. It is easy to note that all the initial data can be expressed in the form of three dimensionless parameters (Re_0, a, $\omega_0=\omega(2R)^2/\nu$) which determine the solution of the nonstationary problem under consideration.

The process of building up the periodic motion occurs sufficiently rapidly. Specific computations show that fluid motion becomes practically periodic after the lapse of 1-2 periods T from the initial time corresponding to the stationary motion.

The computed velocity profiles in both the main and the near-wall flow domains are shown in Fig. 2 for $Re_0=10^5$, $a=0.5$, $\omega_0=10^6$ for three characteristic times in one period of oscillation (i.e., for three different oscillation phases $\varphi=\omega t$) in steady periodic motion. Let us turn attention to the fact that reverse flows are possible in the near-wall region at separate times in unsteady motion, although the instantaneous mean velocity over the section remains positive here during the whole motion ($a<1$). Presented in Fig. 3 are the turbulence energy distributions for the same conditions. Rebuilding of the energy distribution because of the nonstationarity occurs first in the near-wall domain.

Now, let us examine the case when the discharge varies according to the law $Q=Q_1 \sin \omega t$ ($Q_1=\pi R^2 v_1$), i.e., the case when the discharge in the tube varies periodically around the zero value. Computations have shown that for sufficiently high values of ω_0 the maximum value of the velocity occurs at the wall in this case (Fig. 4, $Re_1 \equiv 2v_1 R/\nu = 0.5 \cdot 10^5$, $\omega_0 = 10^4$). Such a picture was observed in the Franke [3] experiments. As the dimensionless frequency ω_0 grows, the velocity maximum continues to be shifted towards the wall and the velocity profile becomes almost uniform in the main part of the section. In this case, a periodic boundary layer exists in the near-wall domain, which does not already enclose the whole tube cross section.

Knowing the velocity distribution, we can calculate the tangential stress on the wall by means of the formula $\tau_0 = -\mu \partial u/\partial r|_{r=R}$ and the energy dissipation by means of (3.2). Represented in Figs. 5 and 6 are the results of their evaluation for $Re_0=10^5$, $a=0.5$, for $\omega_0=10^6$ (curve 1) and for $\omega_0=10^4$ (curve 2), where $\tau_0{}^\circ$ and N° denote the tangential stress and energy dissipation referred to their values in steady motion with discharge Q_0, respectively.

It can be noted in Fig. 5 that the frequency of discharge fluctuation influences both the amplitude and phase shift of the tangential stress.

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